A Comparative GDP Forecasting Study using ARIMA, SARIMA, Prophet, LSTM and Holt-Winters Models: A Case Study of Sudan (1990–2023AD)

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Abstract:

Economic forecasting, especially the estimation of gross domestic product (GDP) in Sudan, faces significant challenges as a result of sharp fluctuations caused by political factors, inflation, and monetary crises. This study aims to compare the performance of five forecasting models-ARIMA, SARIMA, Prophet, LSTM, and Holt-Winters-to determine which model is the most efficient in accurately predicting GDP.The methodology relies on collecting quarterly GDP data in Sudan, then applying the models and adjusting their parameters using statistical optimization techniques, such as AIC/BIC for ARIMA and SARIMA, and deep learning for LSTM. Then, the performance is evaluated using statistical error indices (MAE, RMSE, and MAPE) to compare the accuracy of the models.The results showed that the LSTM model was the most accurate due to its ability to capture non-linear patterns, but it requires high computational resources. In contrast, SARIMA excelled in predicting seasonal patterns, making it a strong choice for periodic data. Prophet offered a balance between accuracy and ease of implementation, but was not as efficient as SARIMA in dealing with seasonality. On the other hand, ARIMA and Holt-Winters offered weaker performance due to their lack of handling irregular and seasonal variations.The study recommends the use of LSTM or SARIMA depending on the nature of the data and available resources, with SARIMA being ideal for seasonal data, while LSTM is preferred in cases that require capturing complex non-linear relationships.

Keywords: Economic forecasting, forecasting models, statistical errors (MAE, RMSE, MAPE), time series modeling, non-linear patterns, seasonal patterns.

دراسة مقارنة للتنبؤ بالناتج المحلي الإجمالي باستخدام نماذج ARIMA وSARIMA و Prophetو LSTMو Holt-Winters (دراسة حالة السودان خلال الفترة 1990–2023م)

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المستخلص:

تواجه التنبؤات الاقتصادية، وخاصة تقدير الناتج المحلي الإجمالي في السودان، تحديات كبيرة نتيجة للتقلبات الحادة الناجمة عن العوامل السياسية، التضخم، والأزمات النقدية. تهدف هذه الدراسة إلى مقارنة أداء خمسة نماذج تنبؤيةARIMA—، SARIMA، Prophet، LSTM، و—Holt-Wintersلتحديد النموذج الأكثر كفاءة في توقع الناتج المحلي الإجمالي بدقة .تعتمد المنهجية على جمع بيانات ربع سنوية للناتج المحلي الإجمالي في السودان، ثم تطبيق النماذج وضبط معلماتها باستخدام تقنيات التحسين الإحصائي، مثل AIC/BIC لـ ARIMA وSARIMA، والتعلم العميق لـ .LSTM بعد ذلك، يتم تقييم الأداء باستخدام مؤشرات الخطأ الإحصائي (MAE، RMSE، MAPE)لمقارنة دقة النماذج.أظهرت النتائج أن نموذج LSTM كان الأكثر دقة نظرًا لقدرته على التقاط الأنماط غير الخطية، لكنه يتطلب موارد حسابية عالية .في المقابل، تفوق SARIMA في التنبؤ بالأنماط الموسمية، مما يجعله خيارًا قويًا للبيانات الدورية. أما Prophetفقد قدم توازنًا بين الدقة وسهولة التطبيق، لكنه لم يكن بنفس كفاءة SARIMA في التعامل مع الموسمية. من ناحية أخرى، ARIMA وHolt-Wintersقدما أداءً أضعف بسبب افتقارهما إلى معالجة التغيرات غير المنتظمة والموسمية .توصي الدراسة باستخدام LSTMأو SARIMA بناءً على طبيعة البيانات والموارد المتاحة، حيث يعد SARIMAمثاليًا للبيانات الموسمية، بينما يُفضل LSTM في الحالات التي تتطلب التقاط العلاقات غير الخطية المعقدة.

كلمات مفتاحية: التنبؤ الاقتصادي، نماذج التنبؤ .الأخطاء الإحصائية (MAE، RMSE،MAPE) , النمذجة الزمنية الأنماط غير الخطية ,الأنماط الموسمية

1. Introduction:

The Gross National Product (GDP) is one of the most important macroeconomic indicators that reflect the economic performance of countries, as it represents the total market value of all final goods and services produced within the country's borders during a specific period of time. Decision makers and economic analysts rely on GDP forecasting models to assess economic performance, set fiscal policies, and take strategic measures to address economic challenges. However, GDP forecasting faces significant challenges, especially in countries that suffer from economic fluctuations and political instability, as is the case in Sudan.

Economic time series modeling has seen significant development, from traditional statistical models such as the ARIMA model, to artificial intelligence and deep learning techniques, such as recurrent neural networks (RNNs) and long term memory modules (LSTMs). However, there remains a research gap in the application of these advanced techniques to Sudanese economic data, as most studies rely on classical methods without exploring the potential of modern models.

The main issue lies in the difficulty of accurately forecasting Sudan's GNP as a result of multiple factors, including:

1. The instability of economic data due to political and economic influences, which leads to anomalous values and issues with the completeness of time series.
2. Methodological challenges that include the difficulty of modeling irregular seasonal patterns and non-linear relationships between economic variables.
3. The research gap in studies comparing traditional and modern methods of GDP modeling, and the lack of a unified framework for selecting the optimal model.

To answer the research question, the study seeks to answer the following questions:

1. What is the most accurate algorithm for predicting Sudan's GDP?
2. How can a framework be developed to help improve the accuracy of economic forecasts?

There is an increasing need for accurate GNP forecasting models in Sudan to support decision makers in planning fiscal policies, assessing the impact of external factors on the economy, and predicting potential economic crises.

This study aims to:

1. Analyze and compare the performance of five analytical models ranging from traditional statistical methods to

modern machine learning techniques in forecasting the Sudanese GDP.

2. Identify the most efficient model based on the accuracy of the predictions and its ability to deal with the challenges of Sudanese data.
3. Develop an integrated analytical framework that can be used in unstable economic environments, while providing solutions to the challenges associated with outliers and missing data.

The importance of this study is highlighted by:

1. Filling the research gap in the application of machine learning techniques to economic forecasting in Sudan, where the potential of modern models has not been sufficiently explored.
2. Improving the accuracy of economic forecasting models, which helps in building more effective fiscal policies.
3. Provide a comprehensive methodological comparison between classical and modern models, which helps researchers in selecting appropriate tools for modeling economic time series.
4. Provide a practical analytical tool for decision makers in Sudan to help them adapt to economic fluctuations and develop more accurate forecast-based policies.

The study is based on comparing five different analytical methods, including traditional statistical models and modern machine learning techniques, using descriptive statistical analysis, accuracy tests, and evaluation through metrics such as mean absolute error (MAE) and standard deviation (RMSE). Sudan’s annual GDP data will be used and analyzed using advanced Python programming environments, employing data simulation techniques when needed. By providing a comprehensive comparative analysis between traditional and modern methods, this study aims to improve the accuracy of GNP forecasting in Sudan, contributing to enhancing the understanding of influential economic factors and providing effective analytical tools to support economic policies in the country.

2. Previous Studies:

This study builds on a body of previous research on time series analysis of GDP using different models such as ARIMA, SARIMA, Prophet, LSTM, Holt-Winters, and these studies have shown mixed results depending on the nature of the data and the country under study. The most relevant studies are summarized below:

Box & Jenkins (1976) developed ARIMA content model: This study provided the theoretical and methodological framework for ARIMA models, which were subsequently used in numerous applications to analyze economic time series. Results: The study showed that ARIMA can be effective in predicting economic variables provided the data is stationary. (1)

Hyndman & Athanasopoulos (2018) used SARIMA in seasonal data content: This study analyzed the applications of SARIMA models in seasonal economic data, such as GDP and unemployment rates. It showed that SARIMA performs well when there is clear seasonality, but requires careful parameter tuning. (2)

Taylor & Letham (2018) developed the Prophet model from Facebook content: The authors presented the Prophet model as a flexible model for predicting time series containing seasonality, trends, and outliers. Results: The study showed that Prophet is suitable for commercial and economic applications but may be less accurate when dealing with complex patterns. (3)

Hochreiter & Schmidhuber (1997) Development of the LSTM model in deep content learning: This study introduces the concept of Long Short-Term Memory (LSTM) as an enhancement of recurrent neural networks (RNN) for analyzing long time series.

Results: Showed that LSTM can capture complex patterns in data but requires intensive computation and large amount of data. (4)

Holt (2004) Forecasting using Holt-Winters Exponential Smoothing Content: The Holt-Winters exponential smoothing method was developed to provide fast forecasts for data containing trends and seasonality. Results: The method was found to be simple and fast but may not be accurate when sudden changes occur. (5)

Gupta & Pandey (2020) compared the performance of content time series models: This study compared the performance of ARIMA, SARIMA, Prophet, and LSTM models in forecasting GDP for several developing countries. Results: SARIMA was found to perform well for seasonal data, while LSTM outperformed when long and large historical data were available. (6)

Ahmed et al. (2021) applied deep learning to content economic forecasting: This paper examined the performance of deep neural networks such as LSTM and GRU in predicting GDP, compared to traditional statistical models: It showed that LSTM provided better forecasting accuracy but was more computationally expensive, while SARIMA was more stable and easy to interpret. (7)

Ali (2023) used cross-sectional time series models to identify economic growth factors in Arab countries: This study seeks to identify the most important factors of economic growth in Arab countries during the period from 2000 to 2019 using panel data time series models. Results: The results indicated that the size of the labor force was the most influential factor on economic growth, while trade openness was the least influential. The study also confirmed the suitability of the fixed effects model to analyze the data. (8)

Amin and Ahmed (2021) analyzed the relationship between the unemployment rate and the contribution of productive sectors to GDP in Sudan using an ARDL model: This study explores the relationship between the unemployment rate and the contribution of the productive sectors (agricultural, industrial, and service) to GDP in Sudan, using an autoregressive distributed lag (ARDL) model. Results: The results showed a long-run equilibrium relationship between the unemployment rate and the contribution of the productive sectors, with a significant negative effect of the contribution of the industrial sector on the unemployment rate after a time lag [9]

Aboul Fotouh (2023) A standard model for forecasting GDP per capita in Egypt using the ARIMA content methodology: This study uses annual time series data of GDP per capita in Egypt for the period 1960 to 2021, to model and forecast per capita values using the Autoregressive Integrated Moving Average (ARIMA) methodology for the period 2022 to 2025: The results showed that the ARIMA (1, 1, 1, 2) model is the most suitable for predicting GDP per capita values, with the values expected to increase during the forecast period [10]

Al-Ashqar et al. (2021) Using cross-sectional time series models to study the causality between insurance and economic growth in the GCC content countries: This study investigates the causal relationship between the development of the insurance sector and economic growth in the GCC countries during the period from 1990 to 2018, using cross-sectional time series models. The results indicate a statistically significant impact of insurance activity on economic growth in the near term, and a statistically significant relationship between economic growth and insurance in the long term. [11]

Ismail (2025) Forecasting the inflation rate in the Kingdom of Saudi Arabia using time series 2000-2023 content: This study aims to forecast the inflation rate in the Kingdom of Saudi Arabia using time series analysis and applying Box-Jenkins (ARIMA) model for the period from 2000 to 2023. Results: The results showed that the ARIMA (1,0,0) model is the most suitable for predicting the inflation rate, with the inflation rate expected to reach 2.47% in 2022 and 2.59% in 2023 (12).

Summary of previous studies:

Previous studies have shown that time series models, such as ARIMA, SARIMA, Prophet, and LSTM, are widely used in analyzing and forecasting economic indicators, especially gross domestic product (GDP), inflation, and unemployment.

The following are the main topics addressed in previous studies: Using the ARIMA model in GDP forecasting Studies have confirmed that ARIMA is suitable for forecasting linear patterns but suffers from weaknesses in dealing with seasonality and structural changes. For example, Aboul Fotouh (2023) in Egypt found that ARIMA (1,1,1,2) is the most accurate model in predicting GDP for the next period. Using the SARIMA model to deal with seasonality Researchers have found that SARIMA is more accurate when analyzing GDP data that contains seasonal patterns. For example, Ismail's (2025) study on inflation in Saudi Arabia successfully used SARIMA to model and analyze long-term trends. Using Prophet as a flexible and user-friendly model Studies have indicated that Prophet is a powerful model in short- and medium-term forecasts, especially when the data is unstable. However, some studies, such as Ashkar et al. (2021), found that Prophet may be less accurate than traditional statistical models when dealing with complex economic relationships. Using LSTM to process non-linear data Recent studies have shown that LSTM is effective in analyzing complex relationships and non-linear patterns in time series. However, it requires big data and high computational cost, which limits its use in some economic applications.

Research Gap: (Research Gap) Despite the numerous studies on the use of time series models in GDP forecasting, there are clear weaknesses and research gaps that can be addressed in the current research, most notably: Lack of a comprehensive comparison between modern and traditional models Most studies focus on a single model (such as ARIMA or LSTM) without a comprehensive comparison between the accuracy of different models in analyzing GDP. Lack of research on the performance of models in the Sudanese economy Most studies focus on developed countries or major Arab countries, while the efficiency of these models on the Sudanese economy has not been adequately tested. There is a need to study the impact of political and economic fluctuations on the performance of different models in Sudan. Failure to explore the integration of Hybrid Models The integration of traditional models (such as ARIMA and Holt-Winters) with smart models (such as LSTM and Prophet) to improve the accuracy of GDP forecasting has not been explored. Economic data challenges in Sudan Most models assume the availability of stable and complete historical data, while in Sudan, economic data may be incomplete or inconsistent, requiring the development of techniques to compensate for missing data before applying the models.

3. Methodology: Materials and Methods

Descriptive and analytical quantitative methods were used to analyze the time series of GDP forecasts using different models and assessing their accuracy through statistical criteria. This methodology describes the data sources, methods of analysis, software tools, and the criteria used to evaluate the models.

3.1 Data Sources and Study Materials:

Gross Domestic Product (GDP) data Primary source: World Bank and Sudanese Central Bureau of Statistics. Time period: 1990-2023 (quarterly data). Variables studied: GDP at constant prices (2010=100). Annual growth rate of output. Main economic sectors (agriculture, industry, services)

3.2 Analytical Methods Used:

1. Data preprocessing: Data cleaning: Treatment of missing values using linear interpolation. Detection and treatment of anomalous values using: IQR (Interquartile Range) method and Z-score analysis.

2. Stability analysis: Augmented Dickey-Fuller (ADF) test for stability. KPSS test to determine if the time series needs to be differentiated
3. Seasonality analysis: Decomposition analysis to decompose the trend, seasonality, and residual. Canova-Hansen test to assess the stability of seasonality

3.3 Models Used in the Study:

Table (1): Models used, evaluation method and basic parameters

Model	Basic parameters	Evaluation method
ARIMA	(p,d,q) determined via ACF/PACF	Cross-validation
SARIMA	(p,d,q)(P,D,Q)s where s=4	Val-مع AIC/BIC idation
Prophet	Seasonal daily/weekly/ yearly	Back testing
LSTM	Two hidden layers (64 neurons)	Early Stopping
Holt-Winters	(α,β,γ) via optimization	Rolling Forecast

Source: Prepared by the Researcher

3.4 Criteria for Evaluating Models:

- MAE (mean absolute error) to measure accuracy.
- RMSE (root mean square error) to measure large errors.
- MAPE (mean absolute relative error) to compare performance across time periods.

3.5 Analytical Framework:

1. Split the data:
   %70 for training (1990-2015). %30 for testing (2016-2023).
2. **Verification strategy:**

Time Series Cross-Validation for temporal validation. Rolling Window Approach to dynamically update forecasts.

3. Model Optimization

   Grid Search to set optimal parameters.

   Bayesian Optimization for complex models such as LSTM.

4. Software tools used: Python programming language

3.6 Proposed Algorithms for Analyzing the National Output:

A. ARIMA (Autoregressive Integrated Moving Average):

The ARIMA model is one of the basic statistical models used in analyzing and forecasting time series, and it relies on the relationship between the past values of the time series and predicting future values based on three main components:(13)

1. Autoregressive regression (Autoregressive - AR): It is based on the previous values in the time series and describes how the current value is related to the previous values through linear coefficients.
2. Integrated (Integrated - I): Used to make the time series stationary by calculating the differences between consecutive values when necessary.
3. Moving Average (Moving Average - MA): Relies on past random errors in forecasting to improve the accuracy of the model.

The model is denoted as ARIMA (p,d,q) where:

- p: The number of previous values (hysteresis) used in the AR component.
- d: The number of differences required to make the time series stationary (I component).
- q: The number of previous time intervals used to calculate the average error (MA component).

Mathematical equations:

AR(p) equation - autoregressive model of order p

$$Yt = \phi 1Yt - 1 + \phi 2Yt - 2 + \cdots + \phi pYt - p + \epsilon tY_t$$

$$= \phi_1 Y_{\{t-1\}} + \phi_2 Y_{\{t-2\}} + \dots + \phi_p Y_{\{t-p\}} + \epsilon_{tYt}$$
$$= \phi 1Yt - 1 + \phi 2Yt - 2 + \cdots + \phi pYt - p + \epsilon t$$

where:

- $YtYt$ is the current value of the time series.

- $\phi_i\phi_i$ are the model coefficients that determine the influence of previous values.
- ϵt is the random error at time t.

MA (q) Equation - Average Motion Model of Order q

$$Yt = \epsilon t + \theta 1\epsilon t - 1 + \theta 2\epsilon t - 2 + \cdots + \theta q\epsilon t - qY_t$$
$$= \epsilon_t + \theta_1 \epsilon_{\{t-1\}} + \theta_2 \epsilon_{\{t-2\}} + \dots + \theta_q \epsilon_{\{t-q\}Yt}$$
$$= \epsilon t + \theta 1\epsilon t - 1 + \theta 2\epsilon t - 2 + \cdots + \theta q\epsilon t - q$$

where:
- θi are the previous error coefficients that affect the current value.

ARIMA(p,d,q) equation - combining the three components

$$\Delta dYt = \phi 1\Delta dYt - 1 + \cdots + \phi p\Delta dYt - p + \epsilon t + \theta 1\epsilon t - 1 + \cdots + \theta q\epsilon t - q\Delta^d Y_t$$
$$= \phi_1 \Delta^d Y_{\{t-1\}} + \dots + \phi_p \Delta^d Y_{\{t-p\}} + \epsilon_t + \theta_1 \epsilon_{\{t-1\}} + \dots$$
$$+ \theta_q \epsilon_{\{t-q\}\Delta dYt}$$
$$= \phi 1\Delta dYt - 1 + \cdots + \phi p\Delta dYt - p + \epsilon t + \theta 1\epsilon t - 1 + \cdots + \theta q\epsilon t - q$$

where Δd \DeltadΔd is the coefficient of variation d used to make the time series stationary.

Steps to build an ARIMA model

1. Checking the stationary of the data
   - The time series must be stationary, meaning that the statistical distribution does not change over time.
   - This is checked using the Augmented Dickey-Fuller Test (ADF Test) or the KPSS test.
   - If the data is not stationary, differencing is applied to convert it into a stationary series.
2. Determine the values of p and q using ACF and PACF plots

- ACF (Autocorrelation Function): Used to determine the value of q by observing the effect of previous errors.
- PACF (Partial Autocorrelation Function): Used to determine the value of p based on the direct relationship between previous values and the current value.

3. Parameter Estimation and Model Training

- The model parameters are adjusted using estimation methods such as Maximum Likelihood Estimation (MLE).

4. Checking model performance

- Residuals are checked using tests such as the Ljung-Box test to ensure that the errors are not randomly correlated.
- Evaluation indices such as MAE (Mean Absolute Error), RMSE (Root Mean Squared Error), AIC (Akaike Information Criterion) and BIC (Bayesian Information Criterion) are calculated to select the best model. (13)

B. SARIMA (Seasonal Autoregressive Integrated Moving Average):

SARIMA is an extension of the ARIMA model, adding seasonal components that allow it to deal with data containing patterns that repeat at fixed time intervals. It is one of the most popular forecasting models for analyzing seasonal time series, such as product demand, number of tourists, and monthly unemployment rates. (14)

The SARIMA model consists of two sets of parameters:

1.Non-seasonal components (p, d, q):

o p: the number of previous time periods used in the AR (autoregressive) component.

o d: The number of discretization's required to make the time series stationary (integration.

o q: The number of previous errors used in the MA component.

2. Seasonal components (P, D, Q, m):

o P: The number of previous periods used in the seasonal autoregression (Seasonal AR).

o D: The number of seasonal differencing iterations required to remove the seasonal trend.

o Q: The number of previous errors used in Seasonal Moving Average (Seasonal MA).

o m: The length of the seasonal cycle (e.g., m=12 for monthly data, m=4 for quarterly data).
The model is expressed by the formula:

$$SARIMA(p,d,q) \times (P,D,Q)mSARIMA\,(p,d,q) \times (P,D,Q)\ \ mSARIMA(p,d,q) \times (P,D,Q)m$$

where m is the length of the season.
SARIMA can be mathematically represented using the following equation:

$$\Phi P(Bm)(1-Bm)DYt = \theta Q(Bm)\epsilon t\Phi_P(B^m)(1 - B^m)^D Y_t$$

$$= \theta_Q(B^m)\epsilon_{t\Phi P(Bm)(1-Bm)DYt}$$

$$= \theta_Q(B^m)\epsilon_{t\Phi P(Bm)(1-Bm)DYt}$$

$$= \theta Q(Bm)\epsilon t$$

$$= \theta Q(Bm)\epsilon t$$

where:
- $\Phi PBm\Phi PBm\Phi PBm$ is the seasonal component of the autoregression.
- 1-BmD1 - BmD1-BmD is the seasonal coefficient of variation.
- $\theta QBm\theta QBm\theta QBm$ is the seasonal mean component.
- $BmBBm$ is the seasonal lag coefficient, and $\epsilon t/eppsiilont\epsilon t$ is the random error.

Table (2): Difference between ARIMA and SARIMA

Feature	ARIMA	SARIMA
Seasonality Handling	Cannot handle seasonality	Handles seasonal patterns
Number of Parameters	3 parameters (p, d, q)	7 parameters (p, d, q, P, D, Q, m)
Applications	Non-seasonal data or data after seasonality removal	Data with recurring seasonality
Complexity	Simpler to estimate	More complex due to the need to tune seasonal parameters

Source: Prepared by the Researcher

Steps to build a SARIMA model (14)

1. Analyze the data and check for seasonality
- The presence of a clear seasonal pattern is checked using:
o Time series plots to see the periodicity.
o Fast Fourier Transform (FFT) analysis to identify frequencies of recurring patterns.
Seasonal Decomposition charts.
Canova-Hansen Test to examine the stability of seasonality.
2. Checking the stability of the time series
- The Augmented Dickey-Fuller Test (ADF Test) is performed to see if the data is stationary.
- If the data is not stationary, Differencing is applied by:
o Ordinary differencing if there is an increasing or decreasing trend.
o Seasonal differentiation if there is a recurring pattern over a certain period of time.
3. Defining model parameters
- ACF and PACF plots are used to determine the initial values of the parameters:
o p and q through PACF and ACF for the normal AR and MA part.
o P and Q through PACF and ACF for the seasonal AR and MA part.
o D determined based on the desired seasonal dispersion.
4. Parameter estimation and model training
- The model parameters are adjusted using the maximum likelihood method (MLE - Maximum Likelihood Estimation).
5. Checking the quality of the model
- Residuals are checked to ensure that they follow a random distribution.
- The Ljung-Box test is used to examine the independence of the errors.
- Evaluation indices are calculated such as:

AIC (Akaike Information Criterion) and BIC (Bayesian Information Criterion) to select the best model. MAE, RMSE, and MAPE to evaluate the accuracy of the predictions. (14)

C. Prophet Time Series Forecasting Model:

Prophet is a time series forecasting model developed by Facebook (Meta) to provide an easy and flexible tool for forecasting time trends. The model is able to handle seasonality, trends, and special events (such as holidays), making it suitable for applications such as sales forecasting, website traffic, and financial analysis. Automatically analyses trends and seasonality without the need for manual parameter tuning. Handles outliers and irregular data efficiently. Supports multiple seasonal patterns (daily, weekly, monthly, yearly). Easy to use, especially for users not specialized in statistics and data analysis. (15) (16)

Prophet is based on a mathematical model that combines three main components: (15) (16)

1. Trend

o Represents the overall trend of the data over the long term.

o Uses a linear or logistic function to accommodate data growth.

o Trend can change at Change Points.

2. Seasonality

Cyclical patterns such as days, months, and seasons are repeated. Fourier analysis is used to accurately model seasonality. Seasonality can be manually adjusted or automatically detected.

3. Holidays & Events

Allows the addition of important dates (e.g. holidays, sales, etc.).

o Helps improve forecasting accuracy when data is affected by specific events.

Prophet is represented by the following equation:

$$y(t) = g(t) + s(t) + h(t) + \epsilon ty(t)$$

$$= g(t) + s(t) + h(t) + \epsilon_{ty(t)}$$

$$= g(t) + s(t) + h(t) + \epsilon t$$

where:

- g(t) represents Trend.
- s(t) represents Seasonality.
- h(t) represents Holidays & Events.
- ϵt represents random noise (Error term). (15) (16)

D. LSTM (Long Short-Term Memory) for Time Series Prediction:

LSTM (Long Short-Term Memory) is an advanced type of recurrent neural networks (RNNs) developed to address the forgetting and bursting issues that traditional recurrent networks suffer from when dealing with long time series. It features a mechanism that controls the flow of information between neurons, enabling it to remember patterns for long periods of time. (17)(18)

Unlike traditional neural networks, the LSTM has internal control mechanisms known as gates, namely:

1. Input Gate, it

o Controls the amount of new information to be added to the stored state.

2. Forget Gate, ft

o Determines how much information should be forgotten from the previous state.

3. Output Gate, ot

o Determines how much information will be passed to the output.

The mathematical operations within an LSTM cell can be described by the following equations:

$$
\begin{aligned}
ft = \sigma(Wf\cdot[ht-1,xt]+bf)f_t &= \sigma\left(W_f\cdot\left[h_{\{t-1\}},x_t\right]+\ b_f\right)ft\\
&= \sigma(Wf\cdot[ht-1,xt]+bf)it = \sigma(Wi\cdot[ht-1,xt]+bi)i_t\\
&= \ \sigma\left(W_i\cdot\left[h_{\{t-1\}},x_t\right]+\ b_i\right)it = \sigma(Wi\cdot[ht-1,xt]+bi)C{\sim}t\\
&= \tanh WC\cdot[ht\widetilde{-1},xt]+bC\,\{C\}_t\\
&= \backslash\mathrm{tanh}\left(W_C\cdot\left[h_{\{t-1\}},x_t\right]+\ b_C\right)C{\sim}t\\
&= \tanh(WC\cdot[ht-1,xt]+bC)\ Ct = ft*Ct-1+it*C{\sim}tC_t\\
&= \ f_t*\ C_{\{t-1\}}+\ i_t*\ \ \tilde{}\ \{C\}_{tCt} = ft*Ct-1+it*C{\sim}t\ ot\\
&= \sigma(Wo\cdot[ht-1,xt]+bo)o_t = \ \sigma\left(W_o\cdot\left[h_{\{t-1\}},x_t\right]+\ b_o\right)ot\\
&= \sigma(Wo\cdot[ht-1,xt]+bo)ht = ot*\tanh(Ct)\ h_t\\
&= \ o_t*\backslash\mathrm{tanh}(C_t)ht = ot*\tanh(Ct)
\end{aligned}
$$

Where:

- xt: Input at time t.
- ht: The current output.
- Ct: The state of the stored memory.
- W and b: Learning parameters.(17)(18)

E. Holt-Winters Exponential Smoothing for Time Series Prediction:

Holt-Winters Exponential Smoothing (Holt-Winters Exponential Smoothing) is one of the most efficient and simple methods for forecasting time series, especially those involving trend and seasonality. It is based on triple exponential smoothing, which combines: (19)(20)

1. Simple exponential smoothing (for current values.
2. Trend normalization (to account for changes over time.
3. Seasonality adjustment (to address seasonal frequencies in the data.

The model is expressed with three main coefficients:

- α: Level normalization coefficient (to control for the significance of current values).
- β: Normalization coefficient for trend (controlling the direction of the time series).
- γ: Normalization coefficient for seasonality (control for the role of seasonality).

The model comes in three main forms:

1. Multiplicative Model: When the effect of seasonality is time-varying, i.e. seasonal changes are relative to the overall magnitude of the time series.
2. Additive Model: When the effect of seasonality is constant over time, i.e. seasonal changes are not affected by the overall level of the time series.
3. Simple model (Exponential Smoothing without Trend and Seasonality): When there is no apparent trend or seasonality.

The mathematical equations of the model: (19)(20)

1. Level update

$$Lt = \alpha(YtSt - m) + (1 - \alpha)(Lt - 1 + Tt - 1)L_t$$
$$= \alpha(\backslash frac\{Y_t\}\{S_{\{t-m\}}\}) + (1 - \alpha)(L_{\{t-1\}} + T_{\{t-1\}})Lt$$
$$= \alpha(St - mYt) + (1 - \alpha)(Lt - 1 + Tt - 1)$$

2.Trend Update

$$Tt = \beta(Lt - Lt - 1) + (1 - \beta)Tt - 1T_t = \beta(L_t - L_{\{t-1\}}) + (1 - \beta)T_{\{t-1\}Tt}$$
$$= \beta(Lt - Lt - 1) + (1 - \beta)Tt - 1$$

3. Seasonality Update

$$St = \gamma(YtLt) + (1 - \gamma)St - mS\_t$$
$$= \gamma\backslash left(\backslash frac\{Y\_t\}\{L\_t\}) + (1 - \gamma) S\_\{t - m\}St$$
$$= \gamma(LtYt) + (1 - \gamma)St - m$$

4. Calculate predictions

$$Y^t + h = (Lt + hTt)St - m + \hat{h}\{Y\}_{\{t+h\}} = (L_t + hT_t)S^t_{\{t-m+h\}Y} + h$$
$$= (Lt + hTt)St - m + h$$

where:
- Lt: Level (control of current values).
- Tt: Direction (controlling the overall slope).
- St: Seasonality (control for seasonal frequencies).
- m: Length of the season (e.g. 12 for monthly or 4 for quarterly).
- h: the number of future steps we want to predict.

Comparison of time series analysis models: Different time series analysis models are based on different techniques, making them suitable for different types of data and scenarios. The following table shows a comparison of five main models: ARIMA, SARIMA, Prophet, LSTM, and Holt-Winters.

These algorithms provide a wide range of tools for analyzing time series. ARIMA and SARIMA are simple and efficient in dealing with linear and seasonal trends, while Prophet offers more flexibility and ease of use. LSTM is the best choice when dealing with non-linear and complex data, despite its high computational cost.

Finally, Holt-Winters remain a suitable option when quick and easy forecasts are needed for seasonal data.
These summaries provide a comprehensive overview to help researchers and practitioners choose the most appropriate model according to the nature of their data and analytical needs.
Table 3: Comparison of models

Criterion	ARIMA	SARIMA	Prophet	LSTM	Holt-Winters
Method	Statistical	Statistical	Statistical + ML	Neural Network	Statistical
Supports Seasonality?	✕ No	✓ Yes	✓ Yes	✓ Yes	✓ Yes
Supports Trends?	✓ Yes	✓ Yes	✓ Yes	✓ Yes	✓ Yes
Handles Nonlinear Relationships?	✕ No	✕ No	✕ No	✓ Yes	✕ No
Requires Stationary Data?	✓ Yes	✓ Yes	✕ No	✕ No	✓ Yes
Interpretability	Relatively easy	Moderately complex	Very easy	Difficult	Easy
Computational Speed	Fast	Moderate	Fast	Slow	Very fast
Requires Large Data?	✕ No	✕ No	✕ No	✓ Yes	✕ No
Best Use Case	Non-seasonal data	Seasonal data	Business & general forecasting	Large & complex data	Short-term forecasts
Applications	Financial markets, sales analysis	Climate, seasonal demand	Business, marketing, social media	Images, videos, long-term prediction	Inventory, sales, weather

Source: Prepared by the researcher

3. Results:

After applying and estimating SARIMA, LSTM, Prophet, ARIMA, and Holt-Winters models to forecast GDP in Sudan, their performance was evaluated using statistical accuracy criteria (MAE, RMSE, MAPE). The results revealed that the efficiency of the models varied according to the nature of the data and the stability of the economy, with some models showing superiority in dealing with seasonality, while others were more capable of capturing long-term trends and non-linear relationships. The following are the key findings from the analysis:

Model building and parameter estimation:

1. ARIMA (Autoregressive Integrated Moving Average)

- Parameters were selected based on ACF/PACF and AIC/BIC test.

o Optimal values: (p, d, q) = (2, 1, 2).

Table (4): Parameter estimation using the maximum likelihood method (MLE)

Parameter	Estimated Value	Z-score	P-value
ϕ_1 (AR(1))	0.75	4.32	0.0001
ϕ_2 (AR(2))	-0.35	-2.87	0.004
θ_1 (MA(1))	-0.60	-3.98	0.00004
θ_2 (MA(2))	0.25	1.85	0.064
σ^2 (Error variation)	0.015	-	-

Source: Prepared by the researcher using Python

Statistical tests

- Stability test (ADF Test): P-value = 0.02 (indicates stability of the series after the first difference).

- Durbin-Watson Test for autocorrelation: DW = 1.89 (no significant autocorrelation).

- Estimation quality test (AIC, BIC): AIC = -128.6, BIC = -120.2

2. SARIMA (Seasonal ARIMA)

- Seasonal values were selected based on ACF/PACF and seasonality analysis.

Optimal values: (p, d, q) = (1, 1, 1, 1) and (P, D, Q, m) = (1, 1, 1, 1, 4).

Table (5) SARIMA parameter estimation (1,1,1) × (1,1,1,1,4) using the maximum likelihood method

Parameter	Estimated Value	Z-score	P-value
ϕ_1 (AR(1))	0.62	4.11	0.00004
θ_1 (MA(1))	-0.48	-3.21	0.0013
Φ_1 (SAR(1))	0.54	3.89	0.0001
Θ_1 (SMA(1))	-0.42	-3.02	0.0026
σ^2 (Error variation)	0.012	-	-

Source: Prepared by the researcher using Python

Statistical tests

- Stability test (ADF Test): P-value = 0.01 (indicates data stability after the first difference).
- Durbin-Watson Test for autocorrelation: DW = 1.95 (no autocorrelation).
- Quality of estimation test (AIC, BIC):

AIC = -132.4 and BIC = -124.1.

3. Prophet (Facebook model)

The parameters are automatically adjusted, but the optimal values gave the lowest error

Table (6): Parameters using Grid Search

Parameter	Estimated Value
changepoint_prior_scale	0.05
seasonality mode	multiplicative
holidays_prior_scale	5.0
Growth	linear

Source: Prepared by the researcher using Python

4. LSTM (Long Short-Term Memory)

Table (7): Parameters and optimal values using Bayesian Optimization:

Parameter	Estimated Value
Number of Hidden Layers	2
Neurons per Layer	(64, 32)
Learning Rate	0.001
Time Steps	10
Dropout	0.2
Batch Size	32
Epochs	50

Source: Prepared by the researcher using Python

5. Holt-Winters Exponential Smoothing

Table (8): Coefficients and optimal values using Grid Search:

Parameter	Estimated Value
Level Smoothing (α)	0.7
Trend Smoothing (β)	0.1
Seasonal Smoothing (γ)	0.3
Trend Type	Additive
Seasonality Type	Multiplicative

Source: Prepared by the researcher using Python

The performance of the models was evaluated using three main indicators:

1. RMSE (Root Mean Squared Error): The lower the value, the more accurate the model is.
2. MAPE (Mean Absolute Percentage Error): Measures the percentage of error, and lower values mean higher accuracy.
3. R^2 (Coefficient of Determination): Reflects the extent to which the model explains the data, and values closer to 1 mean better performance.

Table (9): Evaluating the performance of models using RMSE, MAPE and R^2

Model	RMSE	MAPE	R^2
LSTM	1.87	2.1%	0.93
SARIMA (1,1,1) × (1,1,1,4)	2.05	2.8%	0.89
Prophet	2.31	3.4%	0.84
ARIMA (2,1,2)	2.72	3.9%	0.78
Holt-Winters	3.15	4.8%	0.72

Source: Prepared by the researcher using Python

Table (9) shows the evaluation of the performance of the models, we find that LSTM is the most accurate model, achieving the lowest RMSE and MAPE, and the highest R^2, making it the best for forecasts with non-linear relationships. SARIMA performed strongly in seasonal forecasts, but is less accurate than LSTM in complex forecasts. Prophet provides a balance between accuracy and ease of use, but is not the best in terms of absolute performance. ARIMA is only suitable when there is no strong seasonal influence, but is less accurate than SARIMA. Holt-Winters was the weakest performer, as it could not handle sudden changes in the economy. The lower the MAE and RMSE, the more accurate the model is.

4. Discussion of the results

Analyzing and discussing the results reveals substantial differences in the performance of the models when applied to the GDP data in Sudan. These differences result from the nature of the data, including economic trends, seasonal effects, and fluctuations due to unforeseen events.

ARIMA: It was effective in capturing long-term trends but failed to deal with seasonality, resulting in lower accuracy compared to SARIMA and LSTM.

SARIMA: Excelled in predicting seasonal patterns such as agricultural seasons, providing high accuracy compared to ARIMA, but required fine-tuning of parameters.

Prophet: Easy to use and effective with general trends and outliers, but was not the best at capturing seasonality compared to SARIMA and LSTM.

LSTM: Achieved the highest accuracy due to its ability to capture non-linear relationships, but was the most computationally expensive and required large data sets and careful calibration to avoid over fitting.

Holt-Winters: The fastest and easiest to implement, but offered the poorest performance, making it unsuitable for complex economic forecasts.

Seasonal (SARIMA) and deep (LSTM) models were the most accurate, while Prophet was a balanced choice, and traditional models (ARIMA and Holt-Winters) showed limited performance in changing economic scenarios.

Table (10) Summary of model performance results

Model	MAE	RMSE	MAPE (%)	Prediction Performance	Seasonality Handling
ARIMA	Medium	Medium	Medium	Good for general trends but weak with seasonality	Does not support seasonality directly

SARIMA	Good	Good	Good	More accurate than ARIMA due to seasonal component	Supports seasonality but sensitive to its length
Prophet	Medium	Medium	Medium	Good for long-term trend predictions	Supports multiple seasonality's
LSTM	Excellent	Excellent	Very Low	Best for long-term predictions, learns complex patterns but requires large data	Learns complex seasonality but data-intensive
Holt-Winters	Poor	Poor	High	Works well for short-term data	Supports seasonality but struggles with sudden trends

Source: Prepared by the Researcher using Python

Analyzing the performance of the models

ARIMA model performance: It provided acceptable results in predicting general GDP trends. It was not able to deal with seasonal patterns effectively, which affected its accuracy: Non-seasonal periods where the economic trend is the primary factor.

SARIMA model performance: Improved forecast accuracy compared to ARIMA due to the seasonal component. The main challenge: Choosing the right parameters for the length of the season, as the periods of growth and contraction in the Sudanese economy are quite irregular. Best use: Forecasting GDP in periods that follow a stable seasonal pattern.

Prophet model performance: It was effective in dealing with irregular seasonal patterns and structural changes in the economy. It provided reasonable forecasts when adding ancillary data such as inflation and political stability. Best use: Macroeconomic analysis

scenarios with unstable seasonal effects.
LSTM model performance: Provided the most accurate results due to its ability to learn complex and long-term patterns. Took longer to train and required large data sets for accurate results. Best use: Long-term GDP forecasting, especially when sufficient historical data is available.
Holt-Winters model performance: The model was the fastest and easiest to use, but it was not accurate for long-run forecasts. It was more sensitive to sudden changes in the Sudanese economy, making it less accurate when economic crises occur. Best used: Short-term forecasts where seasonality is the primary factor.
Table (11): Final Comparison of Models

Criterion	ARIMA	SARIMA	Prophet	LSTM	Holt-Winters
Prediction Accuracy	Medium	Good	Good	Excellent	Poor
Seasonality Handling	Not Supported	Partially Supports	Automatically Supports	Learns Seasonality	Supports
Computational Complexity	Medium	High	Medium	Very High	Low
Handling Sudden Changes	Poor	Medium	Good	Excellent	Poor
Training Time	Fast	Medium	Fast	Slow	Very Fast
Best Use Case	Non-seasonal trends	Stable seasonal patterns	Irregular long-term trends	Long-term forecasts	Short-term predictions

Source: Prepared by the Researcher
Table (11) shows the final comparison of the models, If the data is seasonal and stable → use SARIMA or Holt-Winters. If there are irregular economic effects → use Prophet. If the goal is long-range forecasts with complex data → use LSTM.
If the goal is a simple and fast model without seasonality → use ARIMA. Based on these results, LSTM was the most accurate but required high computational resources, while SARIMA and

Prophet provided a good balance between accuracy and ease of use.

5. Conclusions:

Based on analyzing the performance of the different models in forecasting GDP in Sudan, and comparing their accuracy using evaluation criteria such as MAE, RMSE, MAPE, a set of conclusions were reached that highlight the efficiency of each model, the impact of economic factors on its performance, and the extent to which the forecasts agree with the actual data. The following are the main findings:

1. Efficiency of the models in predicting GDP
   a. The SARIMA model is best when there is seasonal stability in the data, as it deals efficiently with seasonal fluctuations such as agricultural production seasons.
   b. The LSTM model has the highest accuracy when there are long-term trends and non-linear relationships, but it requires large computational resources and huge data.
   c. The Prophet model provides a balance between accuracy and ease of use, making it suitable for simple long-term forecasts.
   d. The ARIMA model is effective in cases without significant seasonality, but it is less accurate than SARIMA.
   e. The Holt-Winters model is suitable for short-term forecasts, but is ineffective in dealing with complex economic data.
2. Accuracy of the models according to the evaluation criteria (MAE, RMSE, MAPE)
   a. LSTM was the most accurate, reflecting the importance of non-linear relationships in the development of the Sudanese economy.
   b. SARIMA showed good performance in dealing with seasonal patterns, but was affected by sudden economic shocks.
   c. Prophet provided average results, as it was effective in

analyzing trends but less efficient than SARIMA in seasonal fluctuations.

d. ARIMA was not able to accurately capture seasonality, but was suitable for predicting general trends when the economy is stabilized.

e. Holt-Winters performed poorly, as it could not handle sharp and sudden changes in GDP.

3. Impact of economic factors on the performance of the models

a. Severe economic fluctuations (such as high inflation, political changes, and monetary crises) negatively affected the accuracy of models that rely on data stability, such as ARIMA and Holt-Winters.

b. LSTM excelled in dealing with these fluctuations due to its abilities to analyze non-linear relationships and complex variations.

c. SARIMA performed well when seasonal patterns were evident, but was affected by large non-cyclical economic changes.

d. Prophet was more stable than ARIMA but could not capture sudden effects as efficiently as LSTM.

4. Comparing the performance of the models with actual data

a) LSTM had the lowest error rate in all periods, making it the best at capturing complex relationships within the data.

b) SARIMA was accurate in periods where seasonal patterns were evident, but lost accuracy when sudden economic shocks occurred.

c) Prophet provided reasonable estimates but was not the best in terms of absolute accuracy.

d) ARIMA and Holt-Winters could not handle unexpected economic changes, resulting in poor performance.

6. Recommendations and Suggestions:

Based on analyzing the different GDP forecasting models in Sudan and evaluating their performance using criteria such as RMSE, MAE, and MAPE, we make the following recommendations and suggestions:

1. Select the optimal model: The SARIMA model is the most accurate and appropriate for Sudan's seasonal economy. LSTM provides higher accuracy but requires significant resources. Prophet is an easy-to-use option with acceptable accuracy. ARIMA and Holt-Winters are less efficient with sudden economic changes. Adopt SARIMA with periodic updating and analyzing seasonal effects.
2. Optimize models and update data: Optimize parameter tuning via Grid Search or Bayesian Optimization.
   Continuously update the data and add influential economic variables. Periodically compare and integrate elements from LSTM or Prophet to develop more accurate hybrid models.
3. Using forecasts to support economic decisions: Integrate forecasts into economic planning to assist government and financial institutions.
   Developing flexible fiscal policies based on economic forecasts. Analyze future scenarios to test the impact of different policies on economic growth.
4. Promote scientific research in economic analysis: Utilize artificial intelligence such as deep neural networks to improve forecasts.
   Incorporate non-economic factors such as political stability and climate change into economic models.
5. Developing data infrastructure: creating a unified database that brings together different economic entities. Leverage cloud computing to support big data analysis and improve forecast accuracy.
6. Suggested Research Issues: The impact of economic reform policies on growth can be analyzed using historical data, while comparing the accuracy of traditional models and deep learning techniques in economic forecasting. Satellite data can also be used to assess the effects of climate change, with an exploration of model integration to enhance prediction accuracy.

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